Measuring the magnetic field of an air coil

Objects of the experiments

- Measuring the magnetic field B of a long air coil as a function of the current I.
- Measuring the magnetic field B of a long air coil as a function of the length L and the number N of turns of the coil.

Principles

According to Biot-Savarts law, the magnetic field $\boldsymbol{B}$ generated at the location P by an arbitrary conductor through which a current I is flowing is given by the sum of contributions

$$\mathrm{d}\boldsymbol{B} = \frac{\mu_0}{4\pi} \cdot \frac{I}{r^2} \cdot \mathrm{d}\boldsymbol{s} \times \frac{\boldsymbol{r}}{r} \qquad \text{(I)}$$

$\mu_0 = 4\pi \cdot 10^{-7} \frac{\text{Vs}}{\text{Am}}$: permeability of free space

of the individual parts of the conductor whose length and direction is described by the vector d$\boldsymbol{s}$. In Eq. (I), $\boldsymbol{r}$ is the radius vector from the respective part of the conductor to the point P. That means, calculating the total magnetic field requires evaluation of an integral.

In many cases, this calculation is quite involved and only for conductors with certain symmetries an analytic solution is obtained. In other cases, e.g. when the field of a long coil is calculated, it is much easier to apply Ampères law, which also can be derived from Maxwells equations. It reads

$$\oint_S \boldsymbol{B} \cdot d\boldsymbol{s} = \mu_0 \cdot \int_A \boldsymbol{j} \cdot d\boldsymbol{A} = \mu_0 \cdot I_A \qquad \text{(II)},$$

$\boldsymbol{j}$: current density, I_A: current through the area A,
S: closed boundary curve of the area A

In order to calculate the magnetic field of a long coil, A and S are chosen as shown in Fig. 1. If the coil is sufficiently long, the magnetic field inside the coil is parallel to the axis of the coil and almost vanishes outside the coil, i.e., only on the part S_1 of the boundary curve S will a component of the magnetic field in the direction of the boundary curve be different from zero. Therefore we obtain

$$\oint_S \boldsymbol{B} \cdot d\boldsymbol{s} = \mu_0 \cdot \int_{S_1} \boldsymbol{B} \cdot d\boldsymbol{s} = B \cdot L \qquad \text{(III)}.$$

L: length of the part S_1

Apart from that,

$$I_A = N \cdot I \qquad \text{(IV)}$$

N: number of turns inside A, I: current through the coil

and thus

$$B = \mu_0 \cdot I \cdot \frac{N}{L} \qquad \text{(V)}$$

In this experiment, the magnetic field inside a long coil will be measured by means of an axial B-probe in order to verify the result (V). The probe contains a Hall sensor which is sensitive in the direction parallel to the axis of the probe.

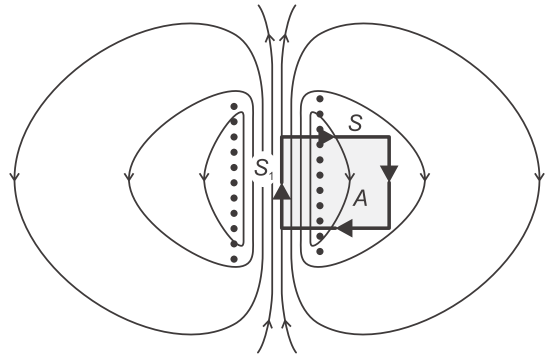

Fig. 1 Calculating the magnetic field of a long coil

Apparatus

1 coil with variable number of turns per unit length	516 242
1 high-current power supply	521 55
1 teslameter	516 62
1 axial B-probe	516 61
1 multicore cable, 6-pole, 1.5 m long	501 16
1 stand for coils and tubes	516 249
1 saddle base	300 11

Setup

The experimental setup is illustrated in Fig. 2.

– Lay the coil with variable number of turns per unit length on the stand for coils and tubes, and connect it to the high-current power supply.
– Connect the axial B-probe to the teslameter via the multicore cable, clamp it with the stand rod from the scope of supply of the probe, and align it so that the Hall sensor **(a)** is located in the centre of the plastic body of the coil.

Carrying out the experiment

a) Measuring as a function of the current *I*:

– Select the measuring range 20 mT at the teslameter, and calibrate the zero with the key Compensation.
– Push the connector sockets **(b, c)** together in a symmetrical way so that the length of the coil is 15 cm (**b**: 12.5 cm, **c**: 27.5 cm).
– Enhance the current *I* in steps of 2 A, and determine the magnetic field *B* in each case; before each new measurement turn the current back to 0 A, and check the zero of the teslameter.

b) Measuring as a function of the length *L*:

– Apply the current *I* = 20 A.
– In order to adjust different coil lengths *L*, pull the connector sockets **(b, c)** apart in a symmetrical way and determine the magnetic field *B* in each case; before each new measurement turn the current back to 0 A, and check the zero of the teslameter.

Measuring example

a) Measuring as a function of the current *I*:

Tab. 1: measuring results for *N* = 30 and *L* = 15 cm

$\frac{I}{A}$	$\frac{B}{mT}$
0	0
2	0.52
4	1.02
6	1.52
8	1.96
10	2.48
12	2.94
14	3.46
16	3.96
18	4.49
20	4.98

b) Measuring as a function of the length *L*:

Tab. 2: measuring results for *N* = 30 and *I* = 20 A

$\frac{L}{cm}$	$\frac{B}{mT}$
8	6.70
10	6.09
15	4.98
20	3.97
25	3.11
30	2.62
35	2.32
40	2.05

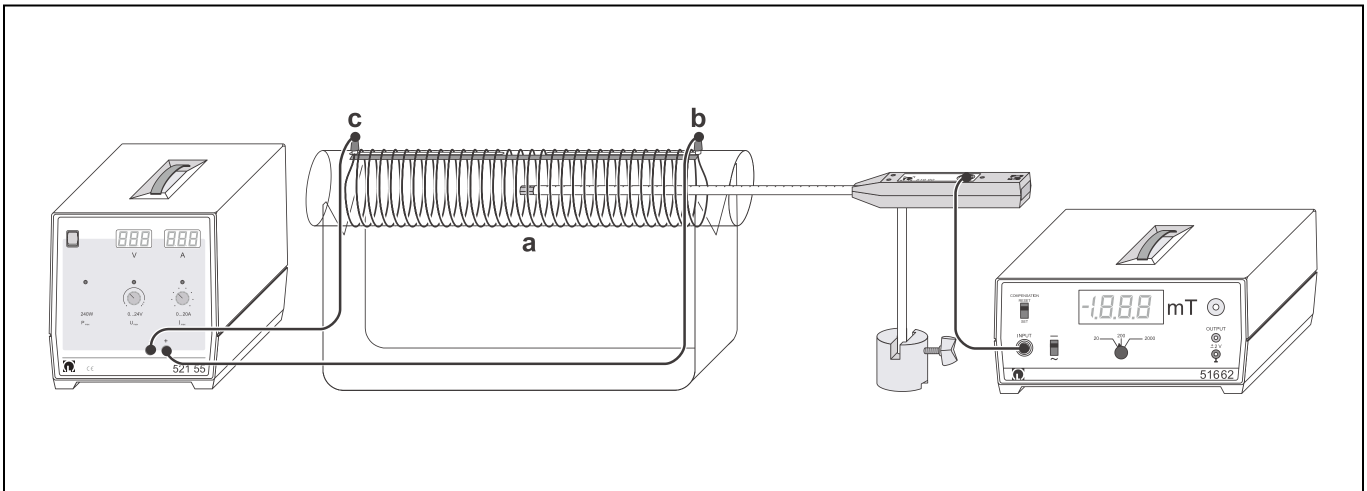

Fig. 2 Experimental setup for measuring the magnetic field of a long coil

Evaluation and results

a) Measuring as a function of the current *I*:

In Fig. 3 the dependence of the magnetic field B on the current I is shown graphically. Within the accuracy of measurement, the measuring results (see Table 1) lie on the straight line drawn through the origin, i.e. the magnetic field B is proportional to the current I.

The slope of the straight line through the origin is $a_1 = 0.248\ \frac{\text{mT}}{\text{A}}$.

According to Eq. (V), the slope of the straight line is

$$a_1 = \mu_0 \cdot \frac{N}{L} = 0.251\ \frac{\text{mT}}{\text{A}}$$

for N = 30 and L = 15 cm.

b) Measuring as a function of the length *L*:

Fig. 4 shows the dependence of the magnetic field on the number of turns per unit length $n = \frac{N}{L}$. For n < 3 cm^{-1}, the values from Table 2 are in good agreement with the straight line drawn through the origin. The deviation for great numbers of turns per unit length n or small lengths L is easily explained as Eq. (II) was calculated for long coils.

The slope of the straight line through the origin is

$a_2 = 2.58$ mT · cm.

According to Eq. (V)

$a_2 = \mu_0 \cdot I = 2{,}51$ mT · cm

for I = 20 A.

In Fig. 5 the dependence of the magnetic field on the length L is shown. For L > 10 cm the measured values are in good agreement with the hyperbola

$$B = \frac{a_3}{L}$$

with $a_3 = 77.5$ mT · cm.

According to Eq. (V) a_3 = 75.4 mT cm for I = 20 A and N = 30.

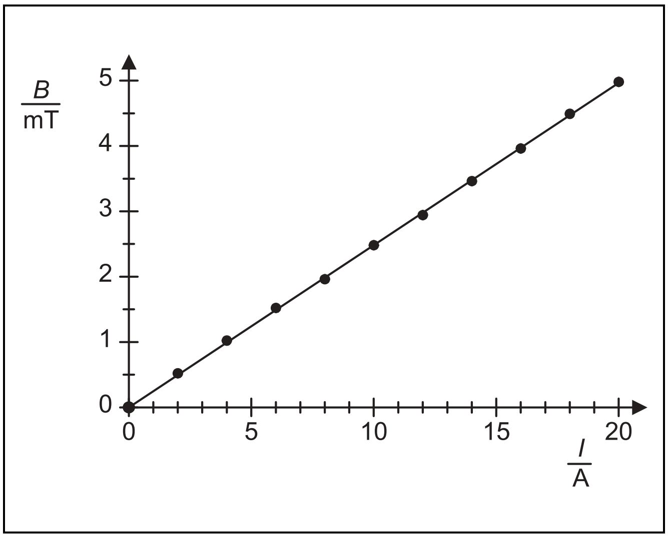

Fig. 3 The magnetic field B in the centre of the air coil as a function of the current I

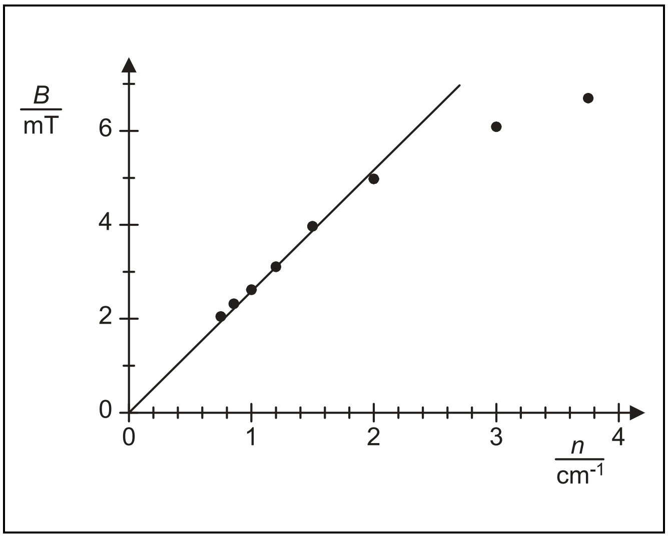

Fig. 4 The magnetic field B in the centre of the air coil as a function of the number of turns per unit length n

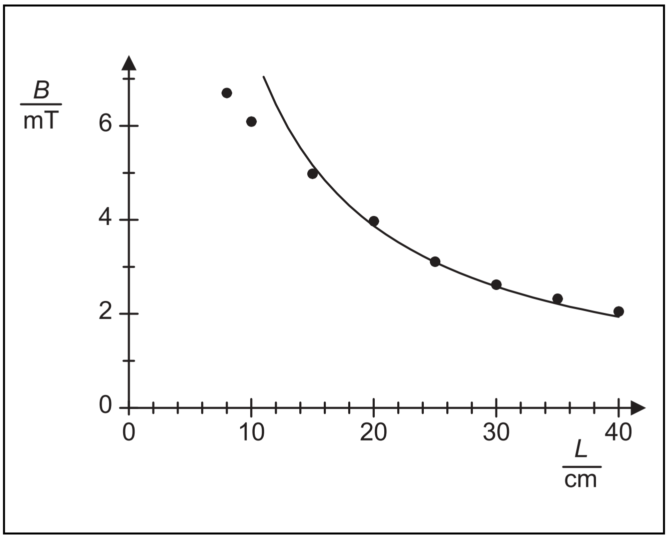

Fig. 5 The magnetic field B in the centre of the air coil as a function of the coil length L at a fixed number of turns N.