Confirming Coulomb's law

Measurement with force sensor and universal measurement instrument physics

Objects of the experiment

- Measuring the force F between two charged spheres in dependence of their distance d.
- Measuring the force F between two charged spheres in dependence of their electric charge.
- Calculation of the electric constant ε_0.

Apparatus

1 Bodies for electric charge, set		314 263
1 Trolley 1, 1,85 g		337 00
1 Precision metal rail, 0,5 m		460 82
1 Force sensor		314 261
1 multicore cable, 6-pole, 1,5m		501 16
1 High voltage power supply, 25kV		521 721
1 Cable for high voltages, 1m		501 05
1 Stand rod with bore holes, 25cm		590 13
1 Saddle base		300 11
1 Elektrometer amplifier		532 14
1 Plug-in power supply 230 V/12V DC		562 791
1 STE Capacitor 1 nF,		578 25
1 STE Capacitor 10 nF,		578 10
1 Voltmeter, up to U = ± 8 V	e.g.	531 100
1 Faraday's cup		546 12
1 Clamping plug		590 011
1 Connecting rod		532 16
1 Stand base, V-shape, small		300 02
1 Stand rod, 25 cm, ø 12mm		300 41
1 Leybold Multi-Clamp		301 01
1 Universal measuring instrument, phys.		531 835
Experimentierkabel		
Connecting leads		

Instead of *universal measuring instrument, physics* CASSY systems like Mobile CASSY 2 (524005) or any other CASSY-Systems are suitable.

Safety note

The high voltage power supply 25 kV is safe corresponding to the safety regulations for laboratory devices. It provides a high voltage that is not touch hazardous. The following safety instructions must be met.

- Read the manual of the high voltage power supply.
- Make sure the high voltage power supply is turned off, before changing the experiment.
- When setting up the experiment, make sure neither not-isolated parts nor wires or plugs may be touched unintended.
- Make sure the control dial of the high voltage power supply is turned to zero before switching the device on
- To prevent sparkovers make sure that there are no conducting objects near any high voltage wires

Preliminary note

Electrostatic experiments are always extremely sensitive to external influences. Therefore it is important for this experiment to keep these external influences (for example air drought) as low as possible. The experiment requires a closed room and low humidity. Charge carriers like the test prod, which is used to charge the spheres, must be far away from the experiment, at least one meter.

If unexpected results occur during the experiment, the plastic sticks of the spheres need to be cleaned. Use distilled water for cleaning only. Conducting salts will be removed this way. Otherwise leakage current may occur and discharge the sphere. This causes massive errors in measuring.

Principles

On two point-like electrical charges Q_1 und Q_2 in a distance d acts a force according to Coulomb's law:

$$F = \frac{1}{4 \cdot \pi \cdot \varepsilon_0} \cdot \frac{Q_1 \cdot Q_2}{d^2}$$

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with

$$\varepsilon_0 = 8.85 \times 10^{-12}\ \frac{\mathrm{As}}{\mathrm{Vm}}$$

Where ε_0 is the electric constant.

The Force has a positive sign, what means that it is repulsive, if both charges have the same sign. If the signs of the charges are different, the sign of the force is negative, what means that it is attractive.

Point-like charges are a simplified model assumption analogue to point-like masses in mechanics. Both do not exist in this form in the real world. Nevertheless this model is close enough to reality, to describe real physics in good precision under certain circumstances. In this experiment conducting spheres will be used. They follow the same physical laws as the point-like charges, as long as they are not too close to each other. If so, they will influence each other's charge distribution and show a significant deviation to the point-charges from the model. To prevent this, it is sufficient to keep the distance d between the centres of the spheres much larger than the sphere's radii r.

The Experiment consists of the parts of which each will investigate one of the following proportionalities:

$$F \sim \frac{1}{d^2} \quad ; \quad F \sim Q_{1,2} \quad ; \quad Q \sim U_{\mathrm{HV}}$$

In all three parts of the experiment it is important to work quickly as soon as any sphere is charged. Else errors will occur due to leakage current which is not completely preventable.

Part I: Proportionality between charging voltage and charge

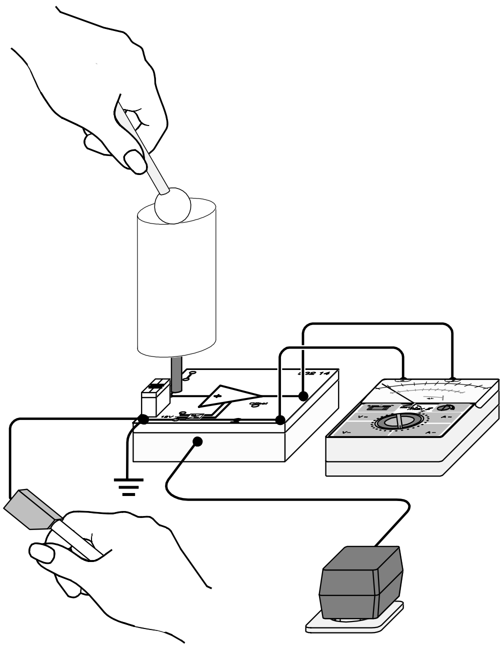

Figure 1: Scheme of the setup for part 1

This first part of the experiment is an important preparation for all following parts of the experiment. The proportionality between charging voltage U_{HV} and charge Q of a sphere charged with this voltage will be investigated. Therefore a electrometer is used. In the end this measurement yields a proportionality-factor that connects both measures.

Setup

For this experiment the high voltage power supply and the electrometer have to be set up. How the electrometer is correctly set up is shown in Figure 1. The 10 nF condenser **(e)** needs to be attached to the top of the electrometer as well as the faraday's cup **(d)** To fit both on the top of the electrometer, a clamping plug must be attached between faraday cup and electrometer. The connecting rod **(f)** must be connected to the electrometer as shown in the sketch.

Then the electrometer is ready for measurement and can be provided with power via the plug in power supply.

Next the high voltage power supply needs to be set up, as shown in Figure 3. First the negative pole must be connected to ground, afterwards a long cable with test prod on the other end needs to be connected to the positive pole. To avoid charging anyone or anything unintended the test prod must be placed in the highest bore of insulated stand rod which is placed in the saddle base.

Finally the power supply must be turned to zero, plugged in and switched on.

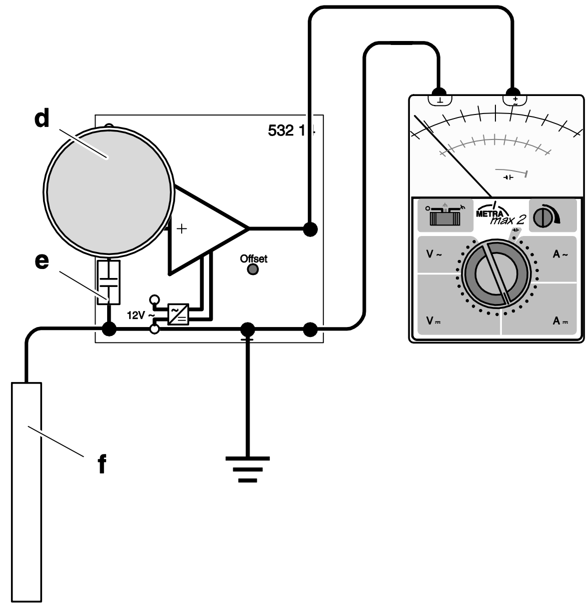

Figure 2: Experiment setup for investigating the relation between charging voltage and charge. Faraday's cup (d), 10 nF-Capacitor (e), Connection rod (f)

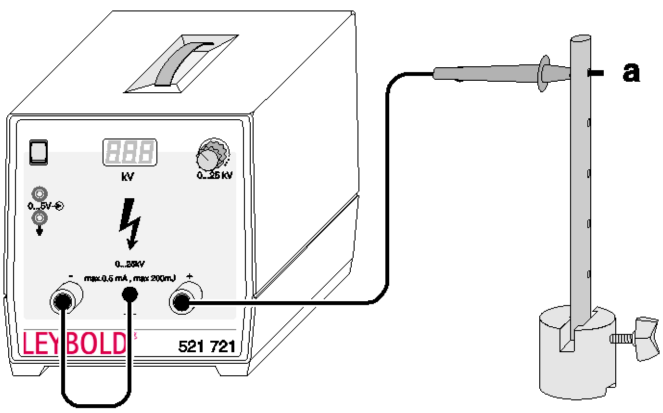

Figure 3: Setup of the high voltage power supply

Carrying out the experiment

1. Hold the connecting rod in one hand.
2. Turn the power supply to 5 kV.
3. Charge the sphere with the test prod (do not touch the sphere directly, only touch the attached insulated plastic stick).
4. Discharge the sphere by touching the Faraday's cup with it.
5. Write down the voltage U_{HV} of the power supply and the voltage from the voltmeter U_{V}.
6. Touch the Faraday's cup and the sphere with the connection stick to discharge both.

Repeat these steps with the following voltages: 7.5 kV; 10 kV; 12.5 kV; 15 kV; 17.5 kV.

Afterwards the high voltage power supply needs to be switched off and its polarity changed. The long cable must be attached to the negative pole, ground to the positive pole. Finally the high voltage power supply must be switched on again, the steps above must be repeated with voltages from 5 kV to 17.5 kV.

Measuring example

The charge Q,comes from U_V and C = 10 nF by

$$Q = C \cdot U$$

U_{HV} / kV	U_{V} / V	Q / nC
5	7,5	8
7,5	1,3	13
10	1,8	18
12,5	2,3	23
15	2,8	28
17,5	3,2	32
-5	-1,2	-12
-7,5	-1,5	-15
-10	-1,9	-19
-15	-2,8	-28
-17,5	3,1	31

Table 1: Measured Data

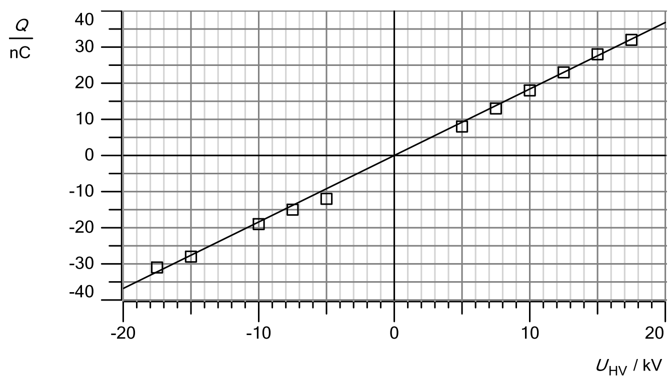

Figure 4: Charge against charging voltage

Evaluation

The slope of the fitted function yields a proportionality factor between charging voltage and charge, with which for every voltage a corresponding charge can be computed. For the example data this factor is about 1.84. That means that the voltage in kV is 1.84 times the charge in nC.

Part II: Relation between charge and Force

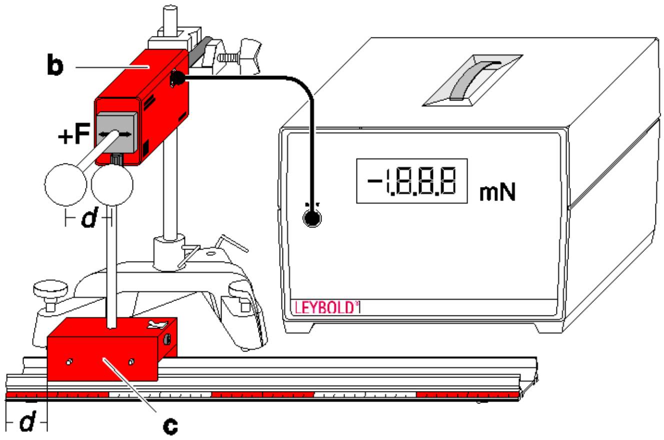

Figure 5 Setup for force measurement

Setup

For this part of the experiment the high voltage power supply and the force sensor will be needed. The high voltage power supply must be set up as in the previous part.

Figure 5 shows how the force sensor needs to be set up. One of the two spheres must be attached to the trolley **(c)**. The other sphere must be attached to the force sensor **(b)**. The force sensor needs to be connected to the universal measuring instrument. Further configurations at the universal measuring instrument are not necessary.

Now the trolley must be put on the precision metal rail. Both needs to be arranged with the result that the surfaces of the spheres have a distance of about 2 mm and that a distance of d = 2 cm can be read from the precision metal rail. Furthermore both spheres must have about the same height.

The Force sensor must be orientated with its negative side to the trolley.

Please note: To get reliable measurements, the force sensor must turned on 30 minutes before the first measurement. Immediately before every measurement the force sensor must be zeroed by pressing the „>>0<<"-button.

Carrying out the experiment

This measurement is about charging one sphere with several different voltages, while the charge of the other sphere and the distance *d* between both spheres is kept constant. The Force is measured as a function of the high voltage.

1. Make sure that $d = 2$ cm
2. Turn the High voltage power supply to the reference voltage (the reference voltage must be the same for the whole part of the experiment.17.5 kV are recommended)
3. Charge one of the spheres (there is no need to choose the same sphere every time) with the reference voltage by touching it with the test prod
4. Turn the high voltage Power supply to 2,5 kV
5. Charge the other sphere
6. Write down force and high voltage

This procedure must be repeated for 5; 7.5; 10; 12.5; 15; 17.5 kV instead of 2.5 kV. Afterwards the poles of the high voltage power supply must be changed and the procedure repeated for all voltages.

Evaluation

In the following example measurement the reference voltage was 17.5 kV. That corresponds to a charge of 32,2 nC. The table with the two columns for voltage and force was extended one column to the right for the charge, which is computed with the proportionality factor from the previous part of the experiment.

U_{HV} / kV	Q / nC	F / mN
2,5	4,6	0,42
5,0	9,2	0,59
7,5	13,8	0,95
10,0	18,4	1,52
12,5	23,0	1,56
15,0	27,6	2,04
17,5	32,2	2,44
-2,5	-4,6	-0,40
-5,0	-9,2	-0,73
-7,5	-13,8	-1,16
-10,0	-18,4	-1,21
-12,5	-23,0	-1,53
-15,0	-27,6	-2,15
-17,5	-32,2	-2,28

Table 2:

To investigate and take a closer look at this relation, the data can be plotted like in Figure 6.

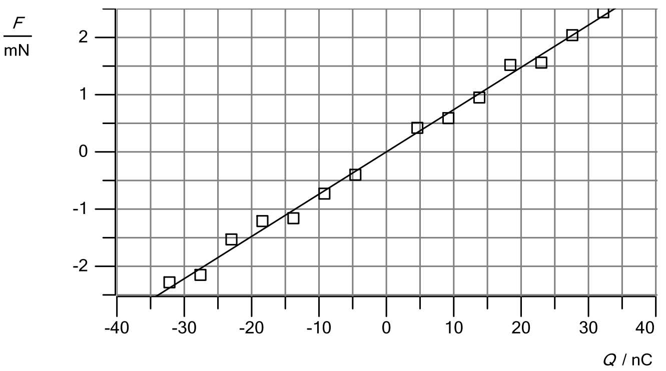

Figure 6: Relation of force and charge

As clearly visible, the relation between force F and charge of the spheres Q is linear.

Part III: Relation between force and distance

Setup

This part of the experiment needs exactly the same setup as the previous one. Therefore the setup can be adopted without any changes.

Carrying out the experiment

To investigate the distance-dependence of the electric force, both spheres will be charged with 17.5kV and same sign. Afterwards the trolley with the sphere must be moved towards the force sensor step by step. For 34; 24; 14; 12; 10; 8; 6 and 4 cm the force and distance must be written down in a table. In detail the measurements consist of the following steps:

1. Put the trolley at 30 cm on the precision metal rail.
2. Make sure the high voltage power supply is adjusted to 17.5 kV
3. Charge both spheres
4. Put the trolley successively to 34; 24; 14; 12; 10; 8; 6 und 4 cm and write down distance and measured force from the force sensor

Evaluation

The example measurement yielded:

d / cm	d^{-2} / cm^{-2}	F / mN
34	0,0009	-0,01
24	0,0017	0,01
14	0,0051	0,40
12	0,0069	0,56
10	0,0100	0,82
8	0,0156	1,21
6	0,0278	2,15
4	0,0625	3,40

To visualize the data, they can be plotted like this:

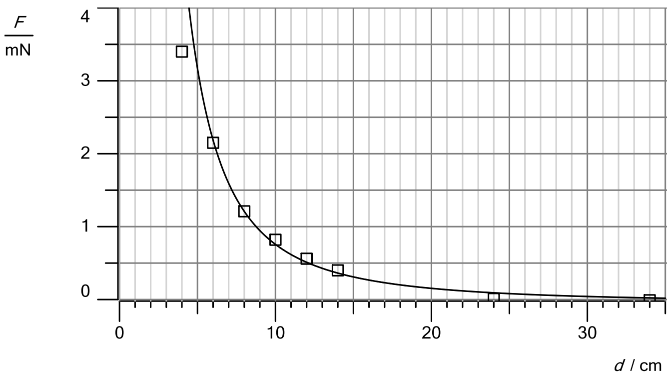

Figure 7: Electric force for different distances

To test the initial assumption

$$F \sim \frac{1}{d^2}$$

and to be able to make a linear fit, the electric force can be plotted versus the inverse squared distance d^{-2} instead of the distance d.

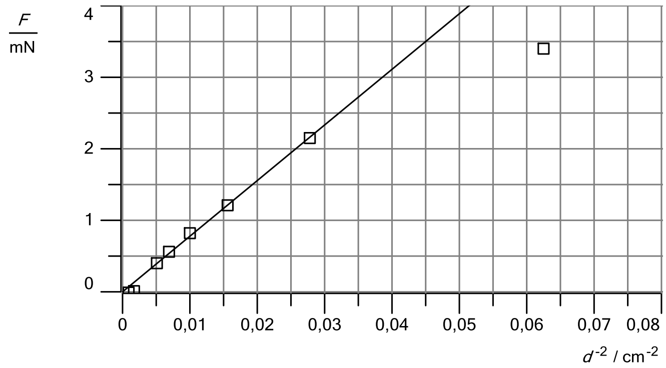

Figure 8: Electric force for different distanced *d* with modified horizontal axis

As clearly visible the last plot shows a linear correlation. That proves the initial assumption. Whereat the last data point was not considered when fitting the straight line, because of the small distance *d*, what could possibly have caused a disturbance in the charge-distribution of the spheres and lead to deviations from the model as mentioned in the preliminary notes.

Determination of ε_0

The symbol $\boldsymbol{\varepsilon_0}$ describes the *permittivity* or *electrical constant*. To determine its value, no further measurement is required. The data already taken are sufficient. Rearranging Coulombs law yields

$$F = \frac{1}{4\,\pi\varepsilon_0}\frac{Q_1 Q_2}{d^2} \qquad \Leftrightarrow \qquad \varepsilon_0 = \frac{1}{4\,\pi}\frac{Q_1 Q_2}{F d^2}$$

with this, ε_0 could be computed from every measured data point of the last two parts of the experiment. But because single measurements may have large statistical errors, it is a much better idea to take some more data into account.

The most elegant way works with the help of Figure 8. The slope of the linear fit line is Fd^2. In the example measurement its value is 77.7. If this is inserted in the rearranged equation from above together with the charge, which is computed via the proportionality factor from the first part of the experiment, follows:

$$\varepsilon_0 = \frac{1}{4\,\pi}\,\frac{32{,}2\ \mathrm{nC}\cdot 32{,}2\ \mathrm{nC}}{77{,}7\ \mathrm{mN\ cm^2}}$$

$$= \ 0.21\ \frac{\mathrm{nC^2}}{\mathrm{mN\cdot cm^2}}$$

$$= \ 10.53\ \times 10^{-12}\,\frac{\mathrm{As}}{\mathrm{Vm}}$$

Reference value: $8{,}85\ \times\ 10^{-12}\ \frac{\mathrm{As}}{\mathrm{Vm}}$