Mechanics

Oscillations

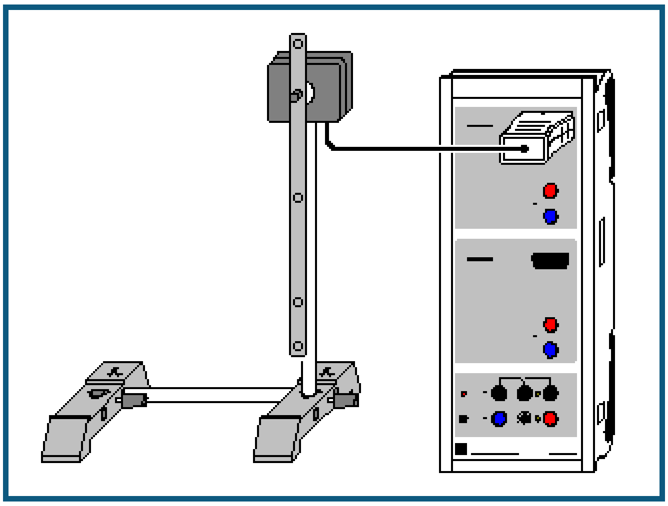
Simple and compound pendulum

Oscillations of a rod pendulum

Description from CASSY Lab 2

For loading examples and settings, please use the CASSY Lab 2 help.

Oscillations of a rod pendulum



can also be carried out with Pocket-CASSY

Experiment description

The equation of motion for a physical pendulum with moment of inertia J, mass m and distance s between fulcrum and center of gravity

$M = J\cdot\alpha'' = -m\cdot g\cdot s\cdot\sin\alpha$

describes for small deflections (sin α≈α) an harmonic oscillation with period of oscillation

$T = 2\pi\cdot\text{sqrt}(J/mgs)$.

For a better understanding, the reduced pendulum length $l_r = J/ms$ is introduced. Then the period of oscillation is

$T = 2\pi\cdot\text{sqrt}(l_r/g)$.

For mathematical pendulums, the entire pendulum mass is concentrated at a single point. It has therefore the moment of inertia $J = ms^2$ and the reduced pendulum length is $l_r = J/ms = s$, which is equivalent to the distance between the pendulum mass (center of gravity) and fulcrum.

A physical pendulum with the reduced pendulum length l_r corresponds to a mathematical pendulum of this length.

In this experiment the reduced pendulum length is determined from the measured period of oscillation and compared with the reduced pendulum length.

Equipment list

1	Sensor-CASSY	524 010 or 524 013
1	CASSY Lab 2	524 220
1	Rotary motion sensor S	524 082
1	Physical pendulum	346 20
1	Stand rod, 25 cm, d = 10 mm	301 26
2	Stand bases MF	301 21
1	PC with Windows XP/Vista/7/8	

Experiment setup (see drawing)

The pendulum is screwed on the axle of the rotary motion sensor.

Carrying out the experiment

Load settings

- Define the zero point in the equilibrium position of the pendulum (→ **0** ← in Settings αA1)
- Deflect the pendulum by approx. 5° only and release
- Start the measurement with . The measurement will stop automatically after 10 s
- Repeat the measurement without the mass or with a further mass added

Evaluation

After several oscillations, the duration of these oscillations can be determined by means of a vertical line and from this the period of oscillation. In the example, the result is T = 0.840 s. With g = 9.81 m/s^2 this results in a reduced pendulum length of l_r= g·T^2/4π^2 = 17.5 cm.

This corresponds well to the calculated (using an approximation) pendulum length l_r of the rod. The moment of inertia of the rod for rotation through the center of gravity is J_S = 1/12·ml^2. The axis of rotation of this pendulum is, however, s = 1/3·l displaced from the center of gravity. According to Steiner's theorem this gives J = J_S + ms^2 = 7/36 ml^2 and lr= 7/36·ml^2 / ms = 7/12·l = 17.5 cm (for l = 30 cm).

Conversely, from the calculated reduced pendulum length and the measured period of oscillation the earth's acceleration due to gravity can also be calculated g = l_r·4π^2/T^2 .

Experimental determination of the reduced pendulum length

If mass m_2 is shifted on the pendulum rod until the period of oscillation T compared to the rod without added mass is unchanged, then the reduced pendulum length l_r will also be unchanged. The position x when the (point) mass is now found increases the moment of inertia of the pendulum by $J_2 = m_2 \cdot x^2$. Because the reduced pendulum length l_r has not been changed, the following applies

$l_r = J/ms = (J + J_2)/(m + m_2)/s'$

with s' being the distance between the new center of gravity and the fulcrum, therefore $s' = (m \cdot s+m_2 \cdot x)/(m+m_2)$. From this you get

$J/ms = (J + m_2 \cdot x^2)/(m \cdot s+m_2 \cdot x) = J/ms \cdot (1 + m_2 \cdot x^2/J)/(1 + m_2 \cdot x/ms)$ or $m_2 \cdot x^2/J = m_2 \cdot x/ms$, therefore

$x = J/ms = l_r$.

The (point) mass is then exactly located at the reduced pendulum length. But, because in reality it has a finite size, this is only an approximation.